

**NASA TECHNICAL
MEMORANDUM**

NASA TM X-52278

N 67-21392
(ACCESSION NUMBER)
10181
(PAGES)
TM-X-52278
(INASA CR OR TMX OR AD NUMBER)

FACILITY FORM 602

(THRU)
1

(CODE)
33

(CATEGORY)

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GAS ENCLOSED BETWEEN PARALLEL PLATES**

by Morris Perlmutter

Lewis Research Center

Cleveland, Ohio

TECHNICAL PAPER proposed for presentation at Ninth National
Heat Transfer Conference sponsored by the American Institute
of Chemical Engineers and the American Society of Mechanical
Engineers Seattle, Washington, August 6-9, 1967

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D.C. • 1967

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ABSTRACT

The problem of transient heat transfer through a monatomic collisionless ideal gas enclosed between parallel walls with wall accommodation coefficients of unity is analyzed for the case of a step change in one wall temperature.

The results indicate that steady state heat transfer condition is approached in an oscillatory manner requiring about 20 to 30 crossings of the atoms across the channel at the mean thermal velocity based on the initial wall temperature, $C_{wi} = (2R_g T_{wi})^{1/2}$, to achieve steady state.

The solution is obtained in terms of an arbitrary quantity Q transported across a unit elemental area some distance X from the channel wall. Results for other transported quantities than energy i.e. density, temperature, can be obtained from the present formulation. The results can be readily extended to an arbitrary change in wall temperature as a function of time.

INTRODUCTION

The interest in rarefied gases has been increasing largely due to the possibility of flight at high altitudes and high speeds. This is because the mean free paths for molecular collisions increases with increasing altitudes. For instance (ref. 1) at an altitude of 62.5 miles the mean free path is 0.1 ft. The mean free path increases to 100 feet at an altitude of 120 miles. Thus, convective heat transfer in a earth satellite open to the atmosphere would be through a rarefied gas.

There has been very little consideration given to transient heat transfer between surfaces enclosing a collisionless gas, i.e. where intermolecular collision of the molecules are sufficiently rare so that they can be neglected. The problem of transient heat transfer from a single plate to a collisionless gas has previously been treated by Yang and Lees (ref. 2) for a step increase in wall temperature. They also included the effects due to a step increase in wall velocity in a direction in the wall plane.

The present analysis considers the effect of a step change in one wall temperature on heat transfer when the collisionless gas is enclosed between two walls. In this case the Yang and Lees solution would apply for early times before the effect due to the second wall would be felt. In the analysis we will determine how long and in what manner the final steady state condition is achieved. If the time for the transient to be completed is a significant part of the time used in a heat transfer situation then this transient effect would have to be included in the analysis.

The present analysis can be readily extended to the case of an arbitrary change in wall temperature with time. In this case if the rate of change in the wall boundary temperature is small compared to the time needed for steady state conditions to be attained then the transient effects also would have to be accounted for in calculating the heat transfer. This assumes that the convective heat transfer is significant compared to the radiant heat transfer. The radiant heat transfer transient for this same problem takes place almost instantaneously so that the transient can be neglected for that case.

The present analysis also serves as a limiting solution to the more general and more difficult problem of transient heat transfer in the case

of a rarefied gas where the intermolecular collisions are sufficiently numerous so that this effect must be included in the analysis.

The present results show the time needed for steady state conditions to be achieved and the behavior of the heat flux as it changes in time from its initial value to its final steady state value. These results are shown for several different values of the ratio of final to initial wall temperatures. Results for both step increases and step decreases in wall temperature are shown. It was assumed that the wall accommodation coefficient was unity and the gas was ideal and monatomic.

The solution is obtained in general form so that various other quantities in the gas such as temperature and density could be obtained as was done in Ref. 2. These results were not carried out because it was felt that they would not be significant in a collisionless gas.

NOMENCLATURE

C	average thermal velocity $(2R_g T)^{1/2}$
D	distance between walls
M	mass of a atom
m_w	mass flux leaving the wall, $(\rho V_2)_w = (\rho_w C_w / 2\pi^{1/2})$
Q	molecular quantity
q	total energy flux
R^2	ratio of wall temperatures, $(T_{w,o}/T_{w,i})$
R_g	gas constant
r	see fig. 1
S	see fig. 1
T	temperature
u	inverse dimensionless velocity, $1/v$

V	molecular velocity
v	dimensionless velocity, v/c_1
X	coordinate normal to wall
θ	dimensionless quantity, $\tau c_i/X$
θ^*	dimensionless quantity, $\tau c_i/(D - X)$
ξ	ratio of densities, ρ/ρ_i
ρ	gas density
τ	time
ψ	dimensionless time, $\tau c_i/D$
$\langle \rangle$	refers to averaged quantity

Subscripts:

$+, -$	to or from wall w_o
i	initial conditions before transient occurs
s	steady state
w_o, w_l	refers to wall w_o or wall w_l

Superscript:

refers to quantity leaving wall at time τ'

ANALYSIS

The problem treated here is the heat transfer through a collisionless gas enclosed between two infinite parallel plates when the temperature of one plate is changed in a stepwise manner. Of prime interest is the net heat transfer between the gas and walls as a function of time from the initiation of the transient until steady state heat transfer conditions are achieved.

The problem of transient heat transfer from a single plate to a collisionless gas has previously been treated by Yang and Lees (ref. 2). They

also included the effects due to a step increase in wall velocity in a direction in its own plane. The results of reference 2 for one wall agree with the present result for the wall that has its temperature suddenly changed at early time before the effects due to the opposite wall are felt.

The analytical model is shown in figure 1. Two plates enclose a collisionless monatomic ideal gas, i.e., the mean free path to collision between molecules in the gas is large compared to the distance across the channel. The plates are assumed to have an accommodation coefficient of 1, that is all the incident molecules come to equilibrium with the wall during collision. Initially the entire system is at temperature T_i . At time $\tau = 0$ there is a step change in the wall w_0 temperature from T_i to T_{w0} . The upper wall temperature remains fixed at $T_{w1} = T_{wi}$.

The analysis of the present problem proceeds as follows: The amount of some molecular quantity $Q(V)$ transported across an elemental area at X at time τ that originated at the lower surface elemental area dA_{w0} at time τ' (fig. 1) can be derived from reference 3 equation (A1) to be

$$(\rho \langle v_2 Q \rangle)_+ = \int_{A_{w0}} \int_V \frac{Q \rho' V^3 \left[\exp - \left(\frac{V}{C'_{w0}} \right)^2 \right] (\cos^2 \phi \, dA_{w0} \, dV)}{\pi^{3/2} C_{w0}^3 S^2} \quad (1)$$

where the mass flux leaving the wall, m'_{w0} is given by $(\rho'_{w0} C_{w0}) / 2\pi^{1/2}$.

Taking a ring element $dA' = 2\pi r \, dr \, dr$ with $S^2 = X^2 + r^2$ equation (1) can be rewritten as

$$(\rho(v_2 Q))_+ = \frac{2}{\pi^{1/2}} \int_S \int_V \left[\frac{Q \rho'_{wo} v^3 \left[\exp - \left(\frac{v}{C'_{wo}} \right)^2 \right] x^2 dS dv}{C'_{wo}^3 S^3} \right] \quad (2)$$

The molecules that reach the elemental area dA at X will be of two kinds. One group will have left the wall before the change in the wall temperature. The second group reaching the elemental area will have left after the transient is initiated. Considering the first group, part of these molecules that reach dA at X at time τ that have velocities less than X/τ for all values of path length S will have left the surface before the change in wall temperature. This term is given by part I in equation (3) where $\theta \equiv \psi/X$. For the molecules with velocities greater than X/τ , only those molecules with path length S greater than τv will have left the surface before the change in wall temperature. This term is given by part II in equation (3).

Now consider the second group of molecules arriving at dA at time τ . This group has left the wall after the change in wall temperature and will consist of molecules with velocities greater than X/τ and path length S less than τv . This term is given by part III in equation (3).

$$\begin{aligned} \frac{2\pi^{1/2}}{\rho_i C_i} (\rho(v_2 Q))_+ &= 4X^2 \left\{ \int_{v=0}^{1/\theta} \int_{S=X}^{\infty} Q \frac{v^3 \exp(-v^2)}{S^3} dS dv \right\} \quad \text{I} \\ &+ \int_{v=1/\theta}^{\infty} \int_{S=\tau v C_i}^{\infty} Q \frac{v^3 \exp(-v^2)}{S^3} dS dv \quad \text{II} \\ &+ 4X^2 \int_{v=1/\theta}^{\infty} \int_{S=X}^{\tau v C_i} \frac{\xi'_{wo}}{R^3} \frac{Q v^3}{S^3} \left[\exp - \left(\frac{v}{R} \right)^2 \right] dS dv \quad \text{III} \end{aligned} \quad (3)$$

Transforming variables, $S = (\psi - \psi')/u$, $v = 1/u$ using the Jacobean gives

$$dS \ dv = \begin{vmatrix} -\frac{1}{u} & -\frac{\psi - \psi'}{u^2} \\ 0 & -\frac{1}{u^2} \end{vmatrix} d\psi' \ du = \frac{d\psi' \ du}{u^3} \quad (4)$$

Combining equations (3) and (4) we obtain

$$\begin{aligned} \frac{2\pi}{\rho_i C_i} \frac{1/2}{(v_2 Q)}_+ &= \left[\int_{u=0}^{u=\infty} \int_{\theta'=-\infty}^{\theta'=\theta-u} + \int_{u=0}^{u=\theta} \int_{\theta'=-\infty}^{\theta'=0} \right] \\ &\times \left[4 \frac{Q(\exp - (u)^{-2})}{u^3(\theta - \theta')^3} d\theta' du \right] + 4 \int_{u=0}^{u=\theta} \int_{\theta'=-\infty}^{\theta'=\theta-u} \\ &\times \frac{\xi'_w Q(\exp - (Ru)^{-2})}{(Ru(\theta - \theta'))^3} d\theta' du \end{aligned} \quad (5)$$

By changing the order of integration this can be written as

$$\begin{aligned} \frac{2\pi}{\rho_i C_i} \frac{1/2}{(v_2 Q)}_+ &= 4 \int_{\theta'=-\infty}^0 \int_{u=0}^{u=\theta-\theta'} \frac{Q(\exp - u^{-2}) du d\theta'}{u^3(\theta - \theta')^3} \\ &+ 4 \int_{\theta'=0}^{\theta'=\theta} \int_{u=0}^{u=\theta-\theta'} \frac{\xi'_w Q(\exp - (Ru)^{-2})}{R^3 u^3 (\theta - \theta')^3} du d\theta' \end{aligned} \quad (6)$$

Similarly the net Q transported from the opposite wall w_1 at X of D across the differential area dA at X is given by

$$\frac{2\pi^{1/2}}{\rho_i C_i} (\rho \langle v_2 Q \rangle)_- = 4 \int_{\theta^* = -\infty}^{\theta^* = 0} \int_{u=0}^{u=\theta^* - \theta^*} \frac{Q(\exp - u^{-2}) du d\theta^*}{(\theta^* - \theta^*)^3 u^3}$$

$$+ 4 \int_{\theta^* = 0}^{\theta^* = \theta^*} \int_{u=0}^{u=\theta^* - \theta^*} \frac{\xi'_{w1} Q(\exp - u^{-2}) du d\theta^*}{(\theta^* - \theta^*)^3 u^3} \quad (7)$$

Then the net quantity Q transported across the elemental area at X is given by

$$(\rho \langle v_2 Q \rangle) = \frac{\rho_i C_i}{2\pi^{1/2}} \left[\rho \langle v_2 Q \rangle_+ + \rho \langle v_2 Q \rangle_- \right] \quad (8)$$

Calculation of mass flux across the channel. - Assuming $Q = 1$ we can obtain the mass flux transported across an elemental area from equations (6) to (8). Carrying out part of the integration in equations (6) and (7) gives

$$\frac{2\pi^{1/2}}{\rho_i C_i} (\rho \langle v_2 \rangle)_+ = 1 - e^{-1/\theta^2} + 2 \int_{\theta^* = 0}^{\theta^* = \theta} \frac{\xi'_{w0} \left[\exp - \frac{1}{R(\theta - \theta')^2} \right]}{R(\theta - \theta')^3} d\theta' \quad (9a)$$

$$\frac{2\pi^{1/2}}{\rho_i C_i} (\rho \langle v_2 \rangle)_- = 1 - e^{-1/\theta^{*2}} + 2 \int_{\theta^{*1} = 0}^{\theta^{*1} = \theta^*} \frac{\xi'_{wl} \left[\exp - \frac{1}{(\theta^* - \theta^{*1})^2} \right]}{(\theta^* - \theta^{*1})^3} d\theta^{*1} \quad (9b)$$

Since at a wall the incident mass flux is equal to the reflected mass flux

$$(\rho \langle v_2 \rangle)_{+w} = (\rho \langle v_2 \rangle)_{-w} \quad (10)$$

Then after the wall transient has occurred, $\tau > 0$, at walls $w0$ and $w1$ we have

$$\frac{(\rho \langle v_2 \rangle)_{+,w1}}{(\rho \langle v_2 \rangle)_{wi}} = \frac{(\rho \langle v_2 \rangle)_{-,w1}}{(\rho \langle v_2 \rangle)_{wi}} = \xi_{wl} \quad (11a)$$

$$\frac{(\rho \langle v_2 \rangle)_{-, w_0}}{(\rho \langle v_2 \rangle)_{w_1}} = \frac{(\rho \langle v_2 \rangle)_{+, w_0}}{(\rho \langle v_2 \rangle)_{w_1}} = R \xi_{w_0} \quad (11b)$$

Combining equations (9a) with (11a) and (9b) with (11b) we obtain a pair of simultaneous integral equations

$$\xi_{w_1} = 1 - e^{-\psi^2} + 2 \int_{\psi' = 0}^{\psi' = \psi} \frac{\xi_{w_0}}{R(\psi - \psi')^3} \exp\left[-\left\{R(\psi - \psi')\right\}^{-2}\right] d\psi' \quad (12a)$$

$$\xi_{w_0} R = 1 - e^{-\psi^2} + 2 \int_{\psi' = 0}^{\psi' = \psi} \frac{\xi_{w_1}}{(\psi - \psi')^3} \exp\left[-(\psi - \psi')^{-2}\right] d\psi' \quad (12b)$$

These are solved by an iterative numerical integration procedure on a high speed electronic computer and the results are shown in figure 2. Also shown in figure 2 are the limiting results for very large times, $\psi \rightarrow \infty$. For this case, equations (12a) and (12b) can be shown to give

$$\xi_{w_1, \psi \rightarrow \infty} = \frac{2R}{1 + R} \quad (13a)$$

$$R \xi_{w_0, \psi \rightarrow \infty} = \frac{2R}{1 + R} \quad (13b)$$

This limiting result for large time agrees with the steady state collisionless solution given in reference 4. For very short times, $\psi \rightarrow 0$ equations (12a) and (12b) become

$$\xi_{w_1, \psi \rightarrow 0} = 1 \quad (14a)$$

$$R \xi_{w_0, \psi \rightarrow 0} = 1 \quad (14b)$$

This limiting result for small times agree with the results of reference 2.

Energy flux from the channel wall to the gas. - For a monatomic gas the

energy transported across the channel is due to translational motion only (ref. 2). The total energy being transferred across the channel is therefore given by

$$q = \rho \left\langle v_2 \left(\frac{v}{2} \right)^2 \right\rangle \quad (15)$$

Then in equations (6) to (8) letting $Q = v^2/2 = C_i^2/2u^2$ we obtain

$$\begin{aligned} \frac{2\pi^{1/2} q}{\rho_i C_i^3} &= \exp\left(-\frac{1}{\theta^* 2}\right) - \exp\left(-\frac{1}{\theta^2}\right) + \frac{\exp\left(-\frac{1}{\theta^* 2}\right)}{2\theta^* 2} - \frac{\exp\left(-\frac{1}{\theta^2}\right)}{2\theta^2} \\ &+ \int_{\theta'=0}^{\theta'=0} \frac{R \xi_{wo} \exp\left[\frac{1}{R(\theta - \theta')^2}\right]}{(\theta - \theta')^3} \left\{ 1 + [R(\theta - \theta')]^{-2} \right\} d\theta' \\ &- \int_{\theta^{*1}=0}^{\theta^{*1}=\theta^*} \frac{\xi_{wl} \exp\left[-\frac{1}{(\theta^* - \theta^{*1})^2}\right]}{(\theta^* - \theta^{*1})^3} \left[1 + (\theta^* - \theta^{*1})^{-2} \right] d\theta^* \quad (16) \end{aligned}$$

To find the total heat flux leaving the walls we write θ and θ^* in terms of X and ψ as given in the nomenclature and take the limits for $X = 0$ and D . This gives at wall wo the result

$$\begin{aligned} \left(\frac{2\pi^{1/2} q}{\rho_i C_i^3} \right)_{X=0} &= R^3 \xi_{wo}(\psi) - 1 + e^{-(1/\psi)^2} \left(1 + \frac{1}{2\psi^2} \right) \\ &- \int_{\psi'=0}^{\psi'=\psi} \frac{\xi_{wl}(\psi') \left[1 + (\psi - \psi')^{-2} \right]}{(\psi - \psi')^3} e^{-1/(\psi - \psi')^2} d\psi' \quad (17a) \end{aligned}$$

Similarly the result at wall wl is given by

$$\left(\frac{2\pi^{1/2} q}{\rho_i C_i^3} \right)_{X=D} = 1 - \xi_{w1}(\psi) - \left(\exp - \frac{1}{\psi^2} \right) \left(1 + \frac{1}{2\psi^2} \right)$$

$$+ \int_{\psi' = 0}^{\psi' = \psi} \frac{R \xi_{wo}(\psi') \{ 1 + [R(\psi - \psi')]^{-2} \}}{(\psi - \psi')^3} \exp \left[- (R(\psi - \psi'))^{-2} \right] d\psi' \quad (17b)$$

These results are shown in figures 3(a) and (b).

The limiting solution for large times, $\psi \rightarrow \infty$ is the fully developed collisionless result and agrees with the value given in reference 6.

$$\left[\frac{\pi^{1/2} q}{\rho_i C_i^3} \right]_{\psi \rightarrow \infty, X=0, X=D} = R(R - 1) \quad (18)$$

For short times, $\psi \rightarrow 0$ we obtain the limiting results for early time for wall wo which agree with reference 2

$$\left(\frac{\pi^{1/2} q}{\rho_i C_i^3} \right)_{\psi \rightarrow 0, X=0} = \frac{R^2 - 1}{2} \quad (19)$$

while at wall wl for early times the heat transfer from the wall to the gas is zero

$$\left(\frac{\pi^{1/2} q}{\rho_i C_i^3} \right)_{\psi \rightarrow 0, X=D} = 0 \quad (20)$$

These results are shown in figure 3.

RESULTS AND CONCLUSIONS

The results for the mass flux leaving the walls are shown in figure 2. Initially the mass flux leaving walls wo and wl remain constant after a

step increase in wall w_0 temperature. Then the mass flux leaving the wall w_1 first starts to change towards the steady state result. Somewhat later the wall w_0 also begins to change towards the steady state result. The results approach the final solution in an oscillatory manner.

The mass fluxes leaving the walls do not change for a short time after the step change in wall temperature because although the wall w_0 temperature changes at $\tau = 0$ the incident mass flux on that wall remains constant and so the mass flux leaving the wall remains constant from continuity. However in the case of the step rise in wall temperature the molecules leaving the heated wall w_0 will travel across to wall w_1 faster because of their higher thermal velocity causing the incident mass flux on wall w_1 to increase as shown in figure 2. This is because not only are the faster moving molecules from the hot wall w_0 now arriving but the slower moving molecules that have left wall w_0 earlier also continue to arrive at wall w_1 . Then since the mass flux incident and therefore also leaving the unheated wall will now be greater, the mass flux incident on the heated wall will become greater and the mass flux leaving the heated wall will now start to rise towards the steady state result. Similar arguments can be used in explaining how the cooled wall temperature case approach the equilibrium mass flux.

The results for the heat flux from the wall to the gas is shown in figure 3(a) for the case where the wall w_0 temperature is raised. It can be seen that immediately after the step change in wall temperature there is a heat transfer from the heated surface w_0 to the gas. The other surface w_1 with its temperature unchanged does not absorb any heat until the molecules

that have left the heated surface reach surface w_1 . Both walls approach the final steady state value asymptotically and are within 1 percent of the value by about dimensionless time $\psi \sim 10$.

The results for a wall temperature decrease is shown in figure 3(b). Heat is absorbed into the walls at the cooled surface w_0 immediately. The other surface w_1 with its temperature unchanged does not transmit any net heat to the gas until the molecules leaving the cooled wall reach it. Both walls then approach the steady state result in a oscillatory manner taking somewhat longer to reach the fully developed result than in the heating case i.e. for $T_{w0}/T_{wi} = 1/16$ 1 percent of fully developed heat transfer is reached in dimensionless time $\psi \sim 30$.

The present formulation of the problem allows the calculations of various properties in the body of the gas such as density and temperature by choosing appropriate properties of Q in equations (6) to (8). This was done in reference 2 for the heat transfer from a single plate. However it was felt for the collisionless gas these properties would not be useful.

To illustrate the length of time needed for the step transient to reach final equilibrium we can run out a sample calculation. Taking time to reach 1 percent of steady state in seconds, temperature in degrees rankine and distance in feet, from the definition of ψ_s we can write time τ_s as

$$\tau_s = \frac{\psi_s D}{(2R_g T_{wi})^{1/2}} = \left[\frac{99,500}{m.w.} T_{wi} \right]^{1/2} \quad (21)$$

Taking the molecular weight, m.w., of air equal to 29, the distance between the plates D as 2 feet and the initial wall temperature T_{wi} as $100^\circ R$ then

if the dimensionless time ψ_s needed to obtain fully developed conditions is set equal to 30 we find from (eq. 21) a time $\tau \sim 0.1$ seconds as about the time needed to achieve thermal equilibrium. This implies that for heat transfer changes on the order of seconds the transient collisionless heat transfer behavior can be significant.

The times for the steady state heat transfer condition to be achieved could be significantly different if the mean free path to collision in the gas was smaller so that intermolecular collisions would have to be accounted for and also if the accommodation coefficient of the surface was not equal 1.

The present results can be extended to arbitrary variations of wall temperature as a function of time by considering the dimensionless temperature ratio R^2 as a function of time in the analysis.

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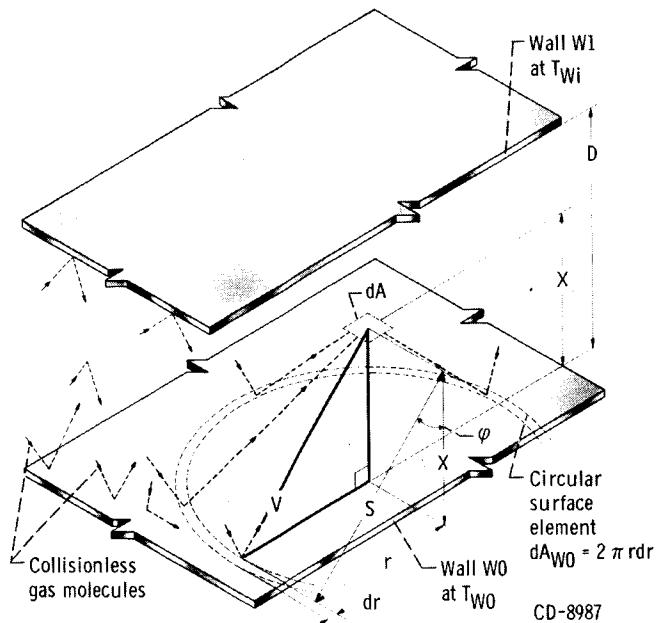


Figure 1. - Analytical model.

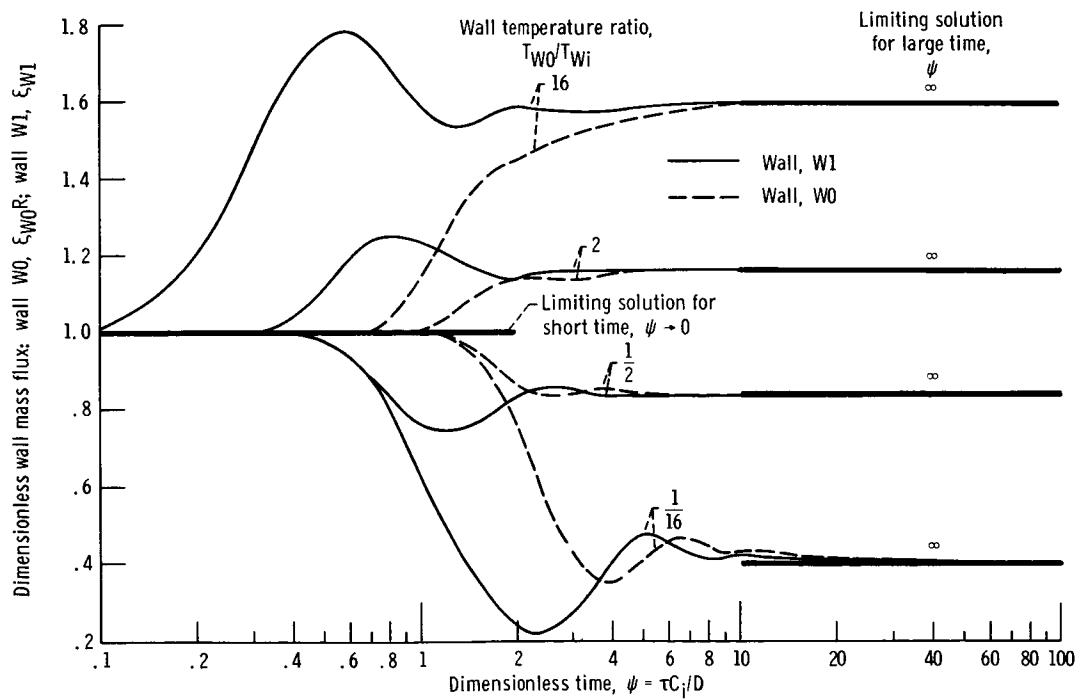
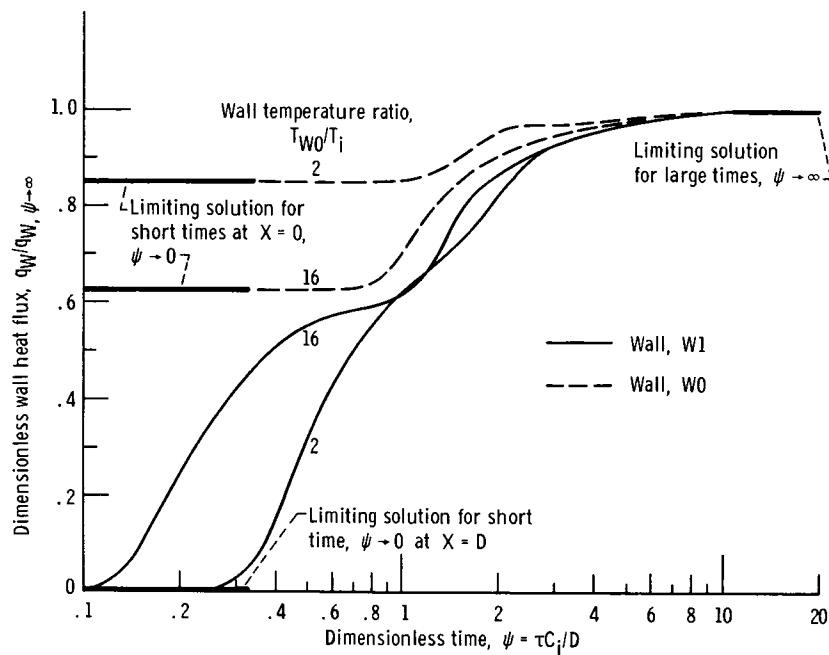
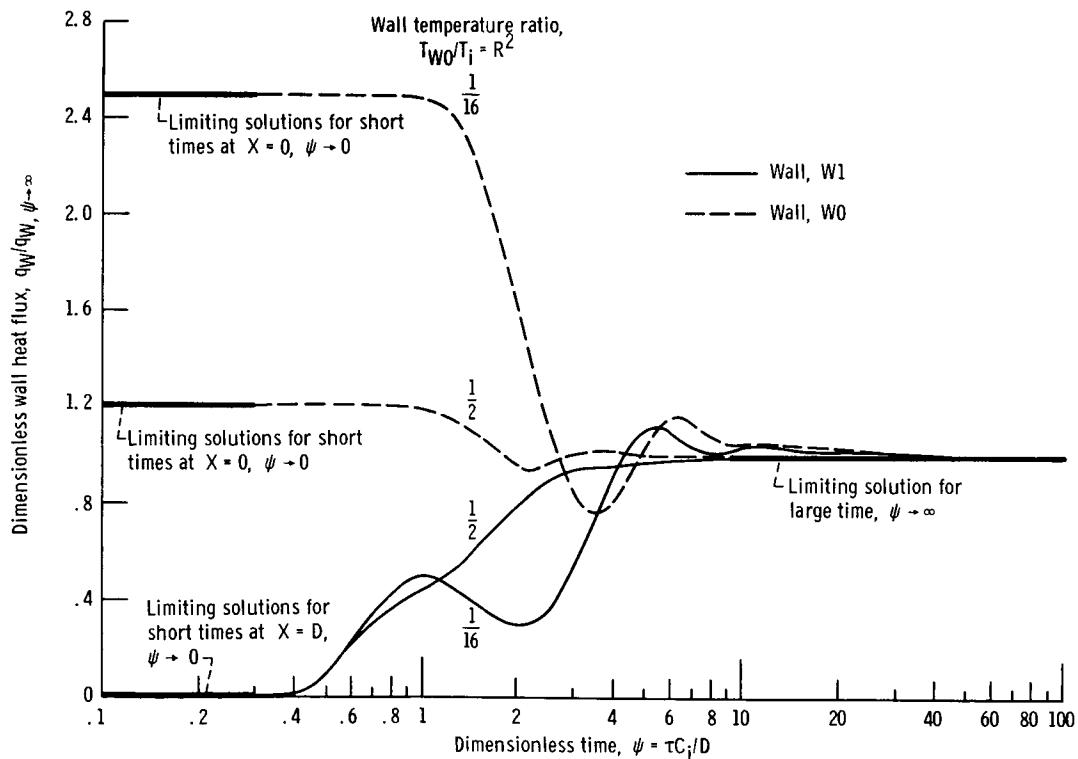


Figure 2. - Dimensionless mass flux leaving the walls for different wall temperature ratios.



(a) Ratios 2 and 16.

Figure 3. - Dimensionless wall heat flux for different wall temperature ratios.



(b) Ratios 1/2 and 1/16.

Figure 3. - Concluded. Dimensionless wall heat flux for different wall temperature ratios.